

The background features abstract, overlapping green geometric shapes, primarily triangles and polygons, in various shades of green, creating a modern and dynamic visual effect.

Strength of Material

Equations of Plane-Stresses Transformation

Plane-Stresses Transformation

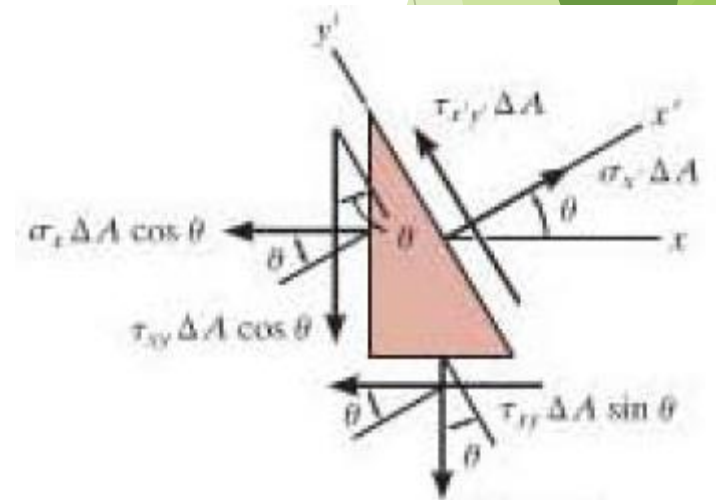
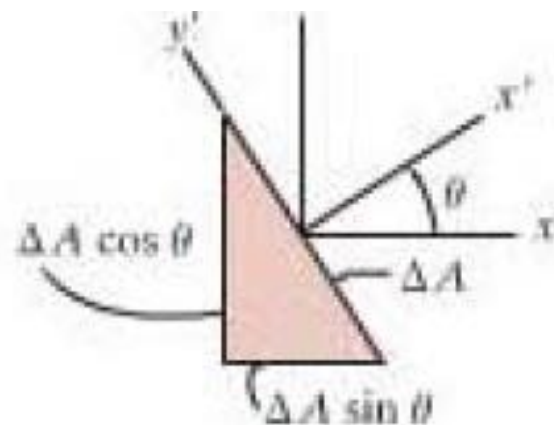
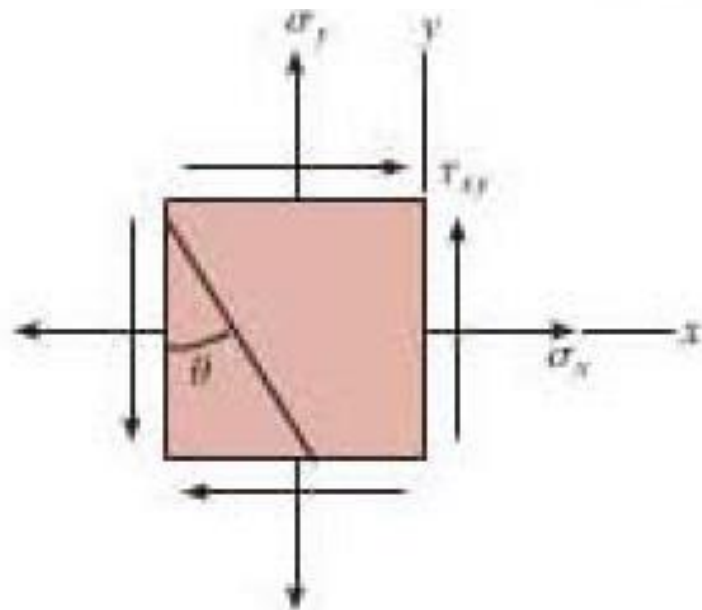
The method of transforming of the normal and shear stress components from the x, y to the $x'' y''$ coordinate axes, as discussed in the previous section, can be developed in a general manner and expressed as a set of stress-transformation equations.

$$+\nearrow \Sigma F_{x'} = 0; \quad \sigma_{x'} \Delta A - (\tau_{xy} \Delta A \sin \theta) \cos \theta - (\sigma_y \Delta A \sin \theta) \sin \theta \\ - (\tau_{xy} \Delta A \cos \theta) \sin \theta - (\sigma_x \Delta A \cos \theta) \cos \theta = 0$$

$$\sigma_{x'} = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + \tau_{xy}(2 \sin \theta \cos \theta)$$

$$+\searrow \Sigma F_{y'} = 0; \quad \tau_{x'y'} \Delta A + (\tau_{xy} \Delta A \sin \theta) \sin \theta - (\sigma_y \Delta A \sin \theta) \cos \theta \\ - (\tau_{xy} \Delta A \cos \theta) \cos \theta + (\sigma_x \Delta A \cos \theta) \sin \theta = 0$$

$$\tau_{x'y'} = (\sigma_y - \sigma_x) \sin \theta \cos \theta + \tau_{xy}(\cos^2 \theta - \sin^2 \theta)$$



These two equations may be simplified by using the trigonometric identities $\sin 2\theta = 2 \sin \theta \cos \theta$, $\sin^2 \theta = (1 - \cos 2\theta)/2$, and $\cos^2 \theta = (1 + \cos 2\theta)/2$, in which case,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

If the normal stress acting in the y' direction is needed, it can be obtained by simply substituting $(\theta = \theta + 90^\circ)$ for θ into Eq. 9-1,

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

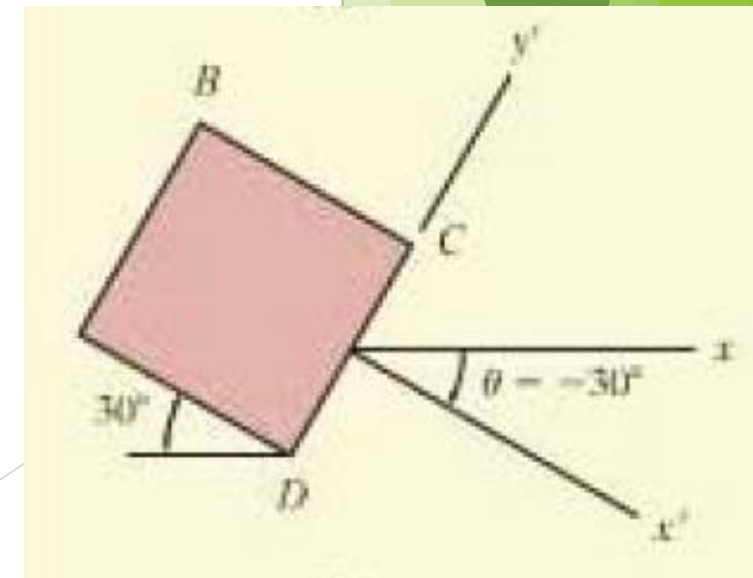
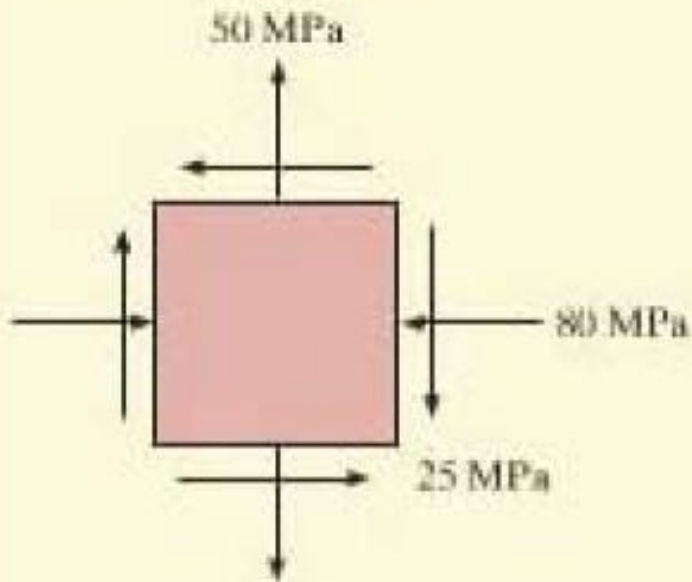
The state of plane stress at a point is represented by the element shown in Fig. 9-7a. Determine the state of stress at the point on another element oriented 30° clockwise from the position shown.

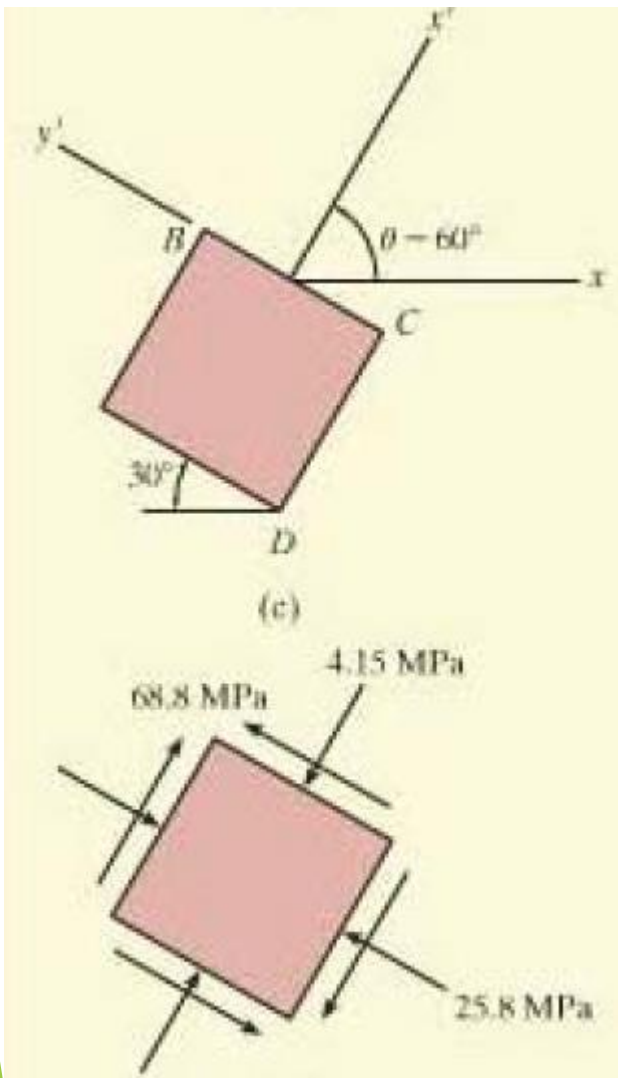
$$\sigma_x = -80 \text{ MPa} \quad \sigma_y = 50 \text{ MPa} \quad \tau_{xy} = -25 \text{ MPa}$$

The plane DC

$$\begin{aligned} \sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{-80 + 50}{2} + \frac{-80 - 50}{2} \cos 2(-30^\circ) + (-25) \sin 2(-30^\circ) \\ &= -25.8 \text{ MPa} \end{aligned}$$

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$$\begin{aligned}\tau_{x'y'} &= -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \\ &= -\frac{-80 - 50}{2} \sin 2(-30^\circ) + (-25) \cos 2(-30^\circ) \\ &= -68.8 \text{ MPa}\end{aligned}$$

Ans.

The plane BC

$$\begin{aligned}\sigma_{x'} &= \frac{-80 + 50}{2} + \frac{-80 - 50}{2} \cos 2(60^\circ) + (-25) \sin 2(60^\circ) \\ &= -4.15 \text{ MPa} \\ \tau_{x'y'} &= -\frac{-80 - 50}{2} \sin 2(60^\circ) + (-25) \cos 2(60^\circ) \\ &= 68.8 \text{ MPa}\end{aligned}$$

Ans.

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