Strength of Material

Equations of Plane-Stresses Transformation

Plane-Stresses Transformation

The method of transforming of the normal and shear stress components from the x, y to the x” y” coordinate axes, as discussed in the previous section, can be developed in a general manner and expressed as a set of stress-transformation equations.

$$+\nearrow \Sigma F_{x'} = 0; \quad \sigma_{x'}\,\Delta A - (\tau_{xy}\,\Delta A \sin\theta)\cos\theta - (\sigma_y\,\Delta A \sin\theta)\sin\theta$$

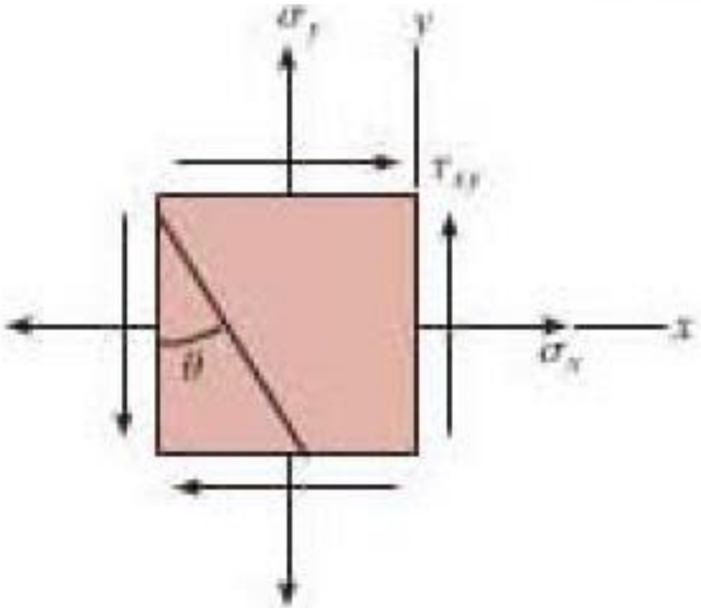
$$- (\tau_{xy}\,\Delta A\cos\theta)\sin\theta - (\sigma_x\,\Delta A\cos\theta)\cos\theta = 0$$

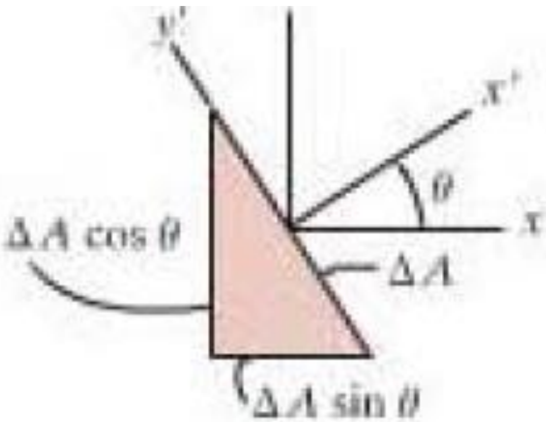
$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau_{xy}(2\sin\theta\cos\theta)$$

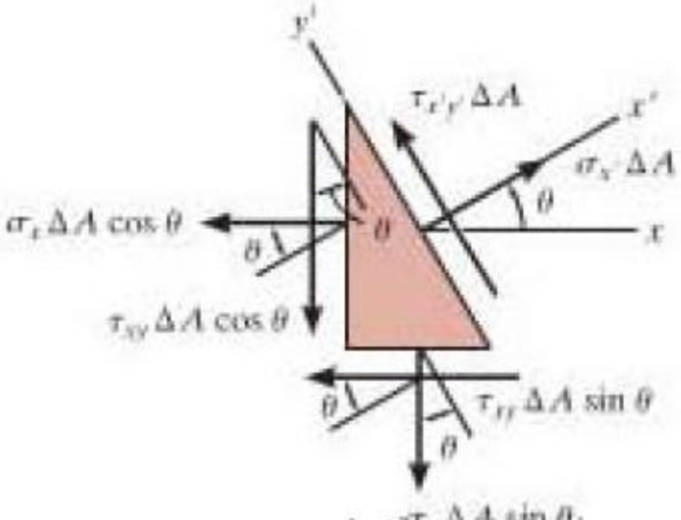
$$+\nwarrow \Sigma F_{y'} = 0; \quad \tau_{x'y'}\,\Delta A + (\tau_{xy}\,\Delta A\sin\theta)\sin\theta - (\sigma_y\,\Delta A\sin\theta)\cos\theta$$

$$- (\tau_{xy}\,\Delta A\cos\theta)\cos\theta + (\sigma_x\,\Delta A\cos\theta)\sin\theta = 0$$

$$\tau_{x'y'} = (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$







These two equations may be simplified by using the trigonometric identities $\sin 2\theta = 2\sin\theta\cos\theta$, $\sin^2\theta = (1 - \cos 2\theta)/2$, and $\cos^2\theta = (1 + \cos 2\theta)/2$, in which case,

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

If the normal stress acting in the y' direction is needed, it can be obtained by simply substituting $(\theta = \theta + 90°)$ for θ into Eq. 9-1.

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

The state of plane stress at a point is represented by the element shown in Fig. 9–7*a*. Determine the state of stress at the point on another element oriented 30° clockwise from the position shown.

$\sigma_x = -80$ MPa $\quad \sigma_y = 50$ MPa $\quad \tau_{xy} = -25$ MPa

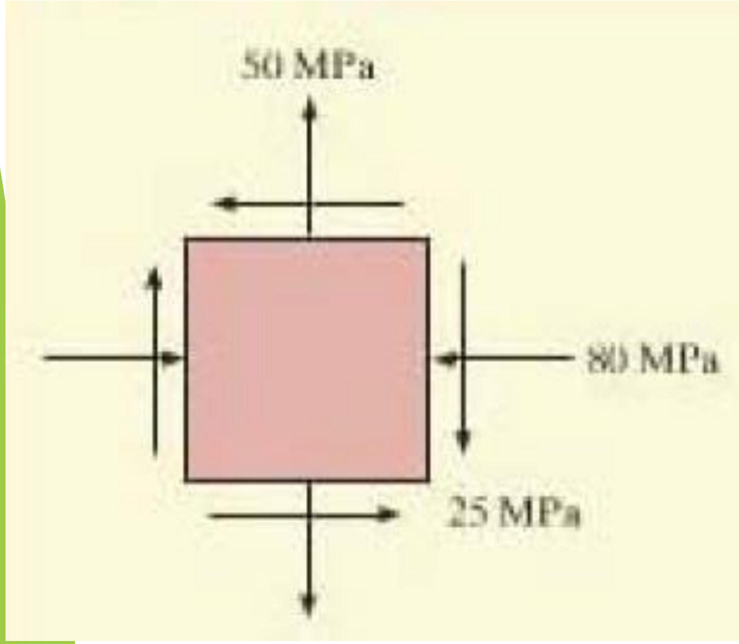
The plane DC

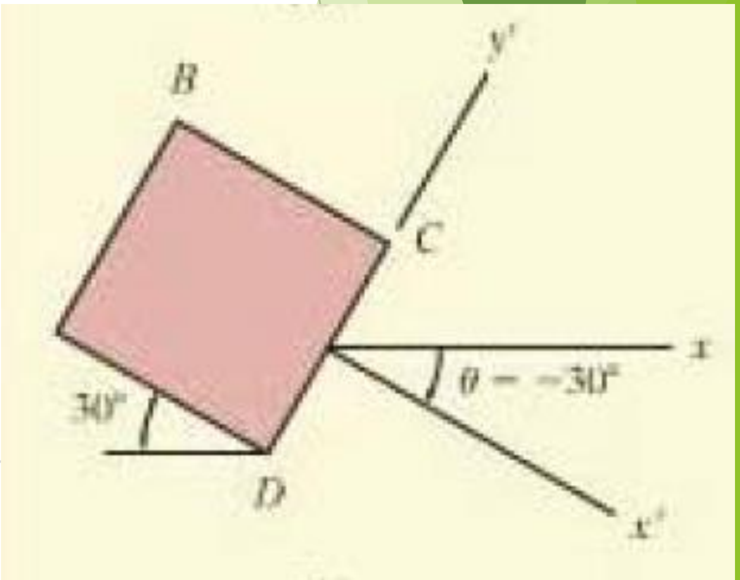
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

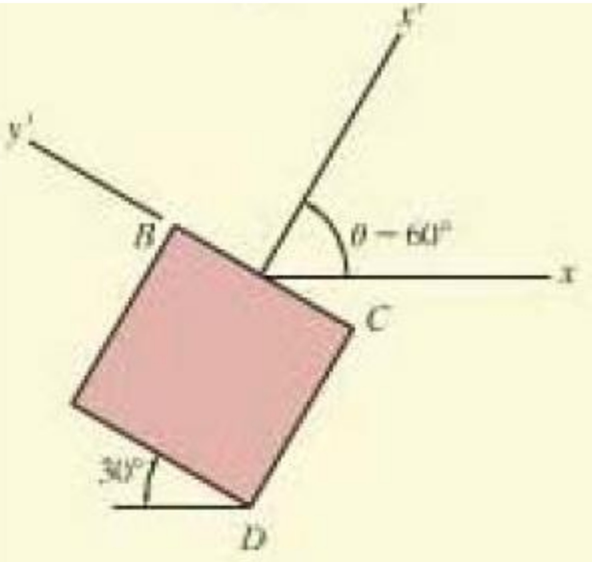
$$= \frac{-80 + 50}{2} + \frac{-80 - 50}{2}\cos 2(-30^\circ) + (-25)\sin 2(-30^\circ)$$

$$= -25.8 \text{ MPa}$$

Ans.







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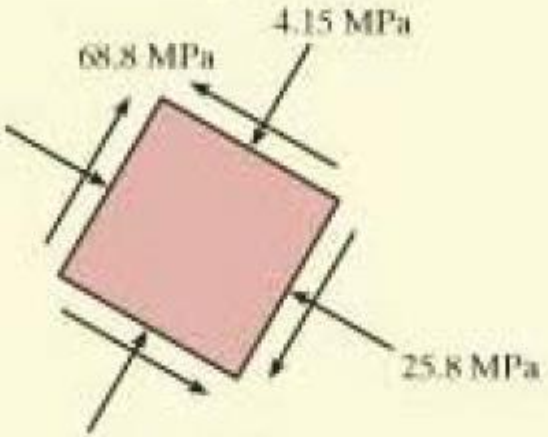

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\frac{-80 - 50}{2}\sin 2(-30^\circ) + (-25)\cos 2(-30^\circ)$$

$$= -68.8\ \text{MPa} \qquad \textit{Ans.}$$

The plane BC

$$\sigma_{x'} = \frac{-80 + 50}{2} + \frac{-80 - 50}{2}\cos 2(60^\circ) + (-25)\sin 2(60^\circ)$$

$$= -4.15\ \text{MPa} \qquad \textit{Ans.}$$

$$\tau_{x'y'} = -\frac{-80 - 50}{2}\sin 2(60^\circ) + (-25)\cos 2(60^\circ)$$

$$= 68.8\ \text{MPa} \qquad \textit{Ans.}$$